# A Stochastic Performance Modeling Technique for Deterministic Medium Access Schemes*

Christoph Lindemann

Technische Universität Berlin
Institut für Technische Informatik
Fachgebiet Prozeßdatenverarbeitung und Robotik
(Real-Time Systems and Robotics)
Franklinstr. 28/29
W-1000 Berlin 10, F. R. Germany

## Abstract

This paper introduces a modeling technique for deterministic medium access schemes to communication channels. The presented approach is based on the extension of Deterministic and Stochastic Petri Nets (DSPNs) by marking-dependent firing delays of deterministic transitions. This extension allows a compact DSPN representation of deterministic medium access schemes. To illustrate the benefit of the described modeling approach a DSPN for CSMA/CD protocols with deterministic collision resolution is presented.

## 1 Introduction

Stochastic Petri net models have been broadly employed for performance analysis of computer and communication systems. Marsan and Chiola introduced Deterministic and Stochastic Petri Nets (DSPNs, [3]) as an extension to Generalized Stochastic Petri Nets. DSPN models include timed transitions associated with either an exponentially distributed firing delay or a deterministic firing delay and immediate transitions firing without a delay. Hence, DSPN models are well suited to represent events which have a constant duration. Under the assumption that in each marking of the DSPN at most one deterministic transition is enabled, the steady-state solution can be determined numerically [3].

Previously, DSPN models could only be employed for modeling small systems (e.g. a LAN with 10 stations [2]) because the applicability of DSPNs have suffered from the high computational effort required by the solution algorithm implemented in the analysis package GreatSPN [6]. Recently, an improved numerical algorithm for solving DSPN models has been developed which significantly reduces the computational effort of the solution [9]. With this improved numerical solution method quite complex DSPN models can be solved with reasonable computational effort.

This paper introduces a modeling technique for deterministic medium access control protocols whose access delay depends on a feature of the communication system which varies during its utilization period. We introduce formulas for calculating the state transition probabilities of the embedded Markov chain and the corresponding conversion factors in case of a deterministic transition with a marking-dependent firing delay. To illustrate the benefit of the described approach a DSPN model for CSMA/CD protocols with deterministic collision resolution is presented. The novel feature of the DSPN model lies in its compact representation of the reservation mode. The stochastic nature of the duration of the selection phase is represented by means of one deterministic transition with a marking-dependent firing delay. These delay values are determined by the average scheduling delay in the reservation mode for each feasible number of stations which may generate a packet. Due to this compact representation of the deterministic access scheme the state space cardinality of the DSPN grows only quadratically with the number of stations. Thus, this model is suitable for evaluating LAN's with a large number of stations.

The remainder of this paper is organized as follows. Section 2 provides a general description of the DSPN modeling approach proposed in this paper. In section 3 is a DSPN model for CSMA/CD protocols with deterministic collision resolution is presented as an application for the proposed approach. Section 4 provides a validation of the quantitative results of the DSPN model against the results obtained from a LANSF [8] simulation program. Finally, concluding remarks are given.

* This work was supported by the Federal Ministry for Research and Technology of Germany (BMFT) and by the German Research Council (DFG) under grants ITR9003 and Ho 1257/2-1, respectively.



0-8186-2755-7/92 $03.00 © 1992 IEEE

## 2 General Description of the Modeling Approach

In this paper we consider deterministic medium access control protocols whose access delay depends on a feature of the communication system which varies during its utilization period (e.g. the number of ready stations at an instant of time). Well known examples of such protocols are the broadcast recognition access method (BRAM [7]) and the multi-accessing tree protocol [4]. This sections provides a general description of the proposed modeling technique. A selection phase and the subsequent transmission is represented in a DSPN model by one deterministic transition with a marking-dependent firing delay. For each feasible number of ready stations the average overhead required by the protocol has to be determined. These average delay values are typically derived by combinatorial formulas.

To illustrate the proposed modeling approach a single-server queueing system with Poisson arrival process, deterministic service time, and limited waiting room is considered. The service time depends on the number of customers who are in the system at the beginning of the current service period. Figure 1 depicts a DSPN model of this queueing system using the ordinary DSPN primitives provided by the software package GreatSPN [6]. The Poisson source is modeled by the place P1 and the exponential transition T0. The state-dependent service process is represented by the places P3,

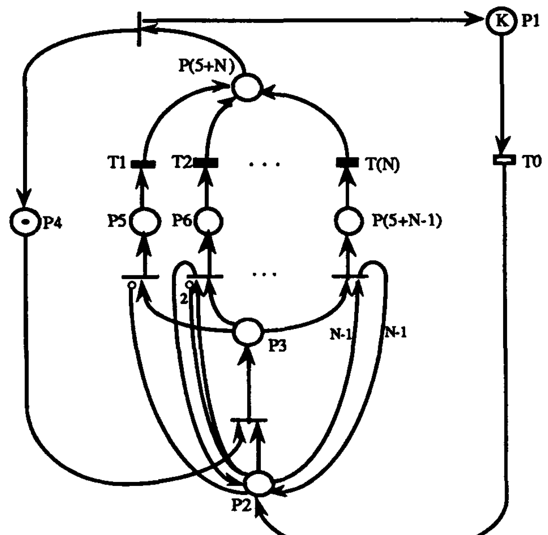


**Figure 1. Detailed DSPN of a M/D/1/K queue with state-dependent service time**

P4, P5,..., P(5+N), the set of conflicting immediate transitions and the set of deterministic transitions. The deterministic transitions T1, T2, ..,T(N) have associated the firing delays of $\psi_1, \psi_2,..., \psi_N$, respectively. The input and inhibitor arcs from place P2 to the immediate transitions are associated with multiplicities such that in case P3 contains a token exactly one of these immediate transitions is enabled. The one which is enabled is determined by the number of tokens in place P2. For example, the left-most will fire, if the place P2 contains no token.

Since the memoryless property does not hold for a deterministic distributed firing delay of a timed transition, additional semantics have to be considered to properly define the association of marking-dependent firing delays to deterministic transitions. In case the marking condition which specifies the marking-dependent delay of a deterministic transition does not remain constant during its enabling interval, an execution policy of its firing process has to be specified [1]. In this paper we consider the case that the firing delay is determined by the marking at the beginning of an enabling period of the deterministic transition. Marking changes during the firing period of a deterministic transition do not influence its current firing delay. DSPN models with deterministic transitions associated with marking-dependent firing delays with this semantic can be efficiently solved by employing similar formulas as introduced in [9].

For each deterministic transition of a DSPN a *subordinated Markov chain* is defined by the firing rates of exponential transitions competitively or concurrently enabled with this deterministic transition and weights of immediate transitions enabled after the firing of the deterministic transition [3]. The matrix $\Delta$ specifies the firing probabilities of such immediate transitions. For each state of the subordinated Markov chain a transient state probability vector has to be calculated at the instant of time corresponding to the firing delay of the deterministic transition. In an ordinary DSPN this instant of time has been fixed for each state of a Markov chain subordinated to a deterministic transition. In case of a marking-dependent firing delay this instant of time may be different for each state. This becomes evident by incorporating in formulas (2) and (3) the i-th component $\psi_i$ of a vector which corresponds to the firing delay of the deterministic transition in marking $M_i$. The generator matrix of the Markov chain subordinated to a particular deterministic transition with a marking-dependent firing delay is denoted by Q. Employing the randomization approach a scalar q and a matrix A are defined as follows.

$$q = 1.02 \cdot \max_{1 \le k \le N} |q_{kk}|$$

$$A = \frac{1}{q} Q + I \quad (1)$$

For a particular deterministic transition the i-th row P(i) of the state transition probability matrix of the embedded Markov chain and the corresponding row C(i) of the conversion matrix are efficiently calculated by.

$$P(i) = \left( \sum_{\nu=L_i}^{R_i} \Phi(\nu) \cdot \beta_i(\nu) \right) \cdot \Delta \quad (2)$$

$$C(i) = \frac{1}{q\psi_i} \sum_{\nu=0}^{R_i} \Phi(\nu) \cdot \left( 1 - \sum_{n=L_i}^{\nu} \beta_i(n) \right) \quad (3)$$

In these formulas $\Phi(\nu)$ denotes the state probability vector of the corresponding discrete-time Markov chain at the instant of time $\nu$ and $\beta_i(\nu)$ denotes the $\nu$-th Poisson probability with parameter $q\psi_i$. The left and right truncation points $L_i = L(q\psi_i, \varepsilon)$ and $R_i = R(q\psi_i, \varepsilon)$ of the summation in formulas (2) and (3) are depending on the pre-defined error tolerance $\varepsilon$. These truncation points are determined by.

$$\sum_{\nu=L_i}^{R_i} \beta_i(\nu) \geq 1 - \varepsilon \quad (4)$$

The function $\Phi(\nu)$ forms the (forward) Chapman-Kolmogorov equation for the discrete-time Markov chain assuming the initial in state $u_i$.

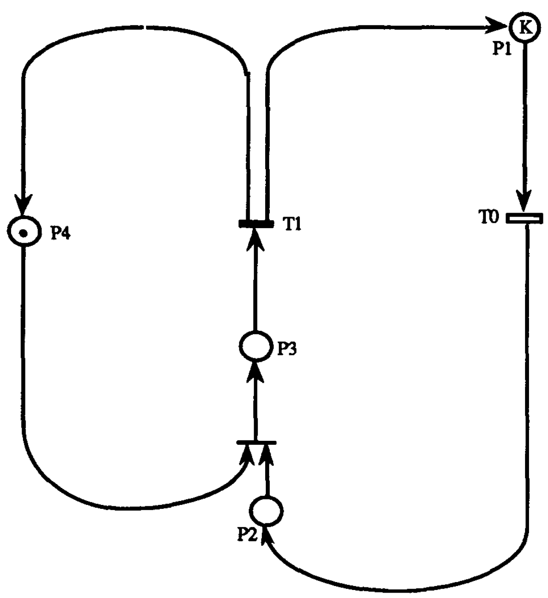


**Figure 2. Compact DSPN of a M/D/1/K queue with state-dependent service time**

The row vector $\Phi(\nu)$ can be efficiently computed by recursive vector-matrix multiplications.

$$\Phi(0) = u_i$$
$$\Phi(\nu + 1) = \Phi(\nu) \cdot A \quad (5)$$

The Poisson probabilities $\beta_i(\nu)$ are calculated recursively as.

$$\beta_i(0) = e^{-q\psi_i}$$
$$\beta_i(\nu + 1) = \beta_i(\nu) \cdot \frac{q\psi_i}{\nu + 1} \quad (6)$$

These formulas allow the compact representation of a M/D/1/K with state-dependent service time depicted in Figure 2. Note, the graphical representation of this DSPN is the same as in case of a M/D/1/K queueing system with fixed service time. The only difference to this case lies in that the firing delay of the deterministic transition T1 depends on the number of tokens which reside in place P2 at the beginning of a firing process of transition T1.

## 3 Modeling CSMA/CD Protocols with Deterministic Collision Resolution

To illustrate the modeling approach two particular CSMA/CD protocols with limited contention are considered. The first protocol is a hybrid of the 1-persistent CSMA/CD with the multi-accessing tree protocol [4]. In case of no collisions the protocol works like the CSMA/CD protocol. Collisions are resolved deterministically by determining the maximum station number among the station numbers of ready stations according to a binary search. Hence, the proposed protocol is referred to as CSMA/CD with binary search (CSMA/CD-BS). As a result of the scheduling phase the station which currently possesses the maximum station number among the ready stations is selected for transmission. This protocol is a modified version of the random multiple access protocols for real-time LAN's proposed by Mierzwa and Wolisz [10].

In the selection phase of the CSMA/CD-BS protocol each station uses the binary representation of its current station number for resolving a collision. Each bit position of this binary representation is assigned to a scheduling slot of length equal to twice of the propagation delay, $2\tau$. Beginning from the highest bit each station with a "1" in the binary representation of its current number starts transmitting in the corresponding slot. If a single station starts transmitting, the selection is finished. All other stations recognize the ongoing transmission in the next scheduling slot due to the carrier sense feature. In case no station starts transmitting the next lower bit of the binary representation of the station number is considered. In case several stations start transmitting a collision is detected by all stations. Stations with a "1" in the bit corresponding to the current slot and a "1" in the next lower bit of the binary representation of their station numbers again start transmitting in the next scheduling slot. This means, only those stations which have started transmitting in the current slot participate further in this scheduling cycle. Stations which have not caused the collision have to wait for the next scheduling cycle. Thus, the selection phase of the CSMA/CD-BS protocol corresponds to finding the maximum of a subset of {1,..,N} corresponding to the stations with ready packets by *binary search*. As a consequence, the selection phase requires at most $\lfloor \log_2 N \rfloor + 1$ scheduling slots. To assure fairness a dynamic rotation of the station numbers is performed. Let N be the total number of station which are numbered by 1, 2,.., N. Assume that the station with number *h* has issued the last successful transmission. Then, during the transmission phase each station with current number *j* ($1 \leq j \leq N$) determines its new station number as $(h - j + 1) \bmod N$.

The second protocol is a hybrid of the 1-persistent CSMA/CD with the broadcast recognition access method [7] and has been introduced by Chen and Li [5]. The difference to the CSMA/CD-BS protocol lies in assigning a particular scheduling slot to each station of the LAN according to its current station number. In the reservation mode each station with a ready packet has to defer the transmission until the beginning of its uniquely associated scheduling slot. Since the scheduling slots are of length $2\tau$, the carrier sense feature ensures that no further collision can occur. To assure fairness a dynamic rotation is performed as described above. This scheduling process can formally be expressed as performing a *linear minimum search* among the station numbers corresponding to ready stations. Thus, the selection phase requires at most N scheduling slots. In the following the protocol is referred to as CSMA/CD with linear search (CSMA/CD-LS). Its detailed description is given in [5].

We consider a local area network (LAN) consisting of N statistically identical stations connected by a bidirectional bus. The end-to-end propagation delay on the bus is denoted by $\tau$. Since the class of protocols is typically employed for control applications of technical processes, a fixed packet length is considered. Each station generates a packet according an exponential distribution with rate $\lambda$. We assume that the stations have a single buffer. Thus, a packet is considered lost, if it is generated by a station before the previous one has been successfully transmitted.

A DSPN model for the CSMA/CD protocols with deterministic collision resolution can be derived from the compact DSPN model for the 1-persistent CSMA/CD protocol introduced by Marsan, Chiola, and Fumagalli [2] by modifying the collision resolution part. The DSPN model takes into account the jamming delay, the finite number of stations and their distribution along the bus. Figure 3 shows the DSPN model for the considered protocols. Each of the N tokens in place P1 depicted in Figure 3 represents a station of the LAN to be modeled. The thinking time of a station is represented an exponentially distributed firing delay with rate $\lambda$ which is associated with the transition T1. Since the thinking times of individual stations are independent from each other, the transition T1 has associated the enabling dependence *infinite-server*. All other timed transitions have associated the enabling dependence *single-server*. The firing of the transition T1 moves a token from place P1 to P2 representing that a station has generated a packet and is competing for the bus. If the bus is sensed idle (place P3 contains a token) the immediate transition t2 fires and puts a token in the places P4 and P5. The sojourn time of this marking represents the vulnerable period of a transmission in the contention mode. The deterministic transition T4 truncates the firing time of the exponential transition T3 in order to approximately represented the triangular distribution of the propagation delay between pairs of stations as introduced in [2]. Suppose, T3 fires putting the token from place P4 to P6 before another station has generated a packet (T1 has not fired again). Then, the immediate transition t6 is enabled. Its firing removes the token from place P6 modeling the termination of the vulnerable period of a transmission in the contention mode. Another possibility in the situation mentioned above is that the deterministic transition T4 fires next moving the token from place P5 to P7. Then, the firing of the immediate transition t5 corresponds to the firing of transition t6 and represents the end of the vulnerable period. In the current marking only the timed transitions T1 and T7 are enabled. The firing delay of the deterministic transition T7 models the remaining transmission delay after the signal has propagated to each station of the LAN. The firing of transition T7 represents the termination of a successful transmission. The station is put back into the thinking state (the token is moved to P1) and the system switches to the reservation mode represented by putting a token in place P15. During the transmission phase (a token in place P7) firings of the exponential transition T1 represent subsequent transmission requests. Such packets are queued (hold in place P2) because the bus is sensed busy (transition t8 is no longer enabled). After the next successful transmission these packets will be processed in the reservation

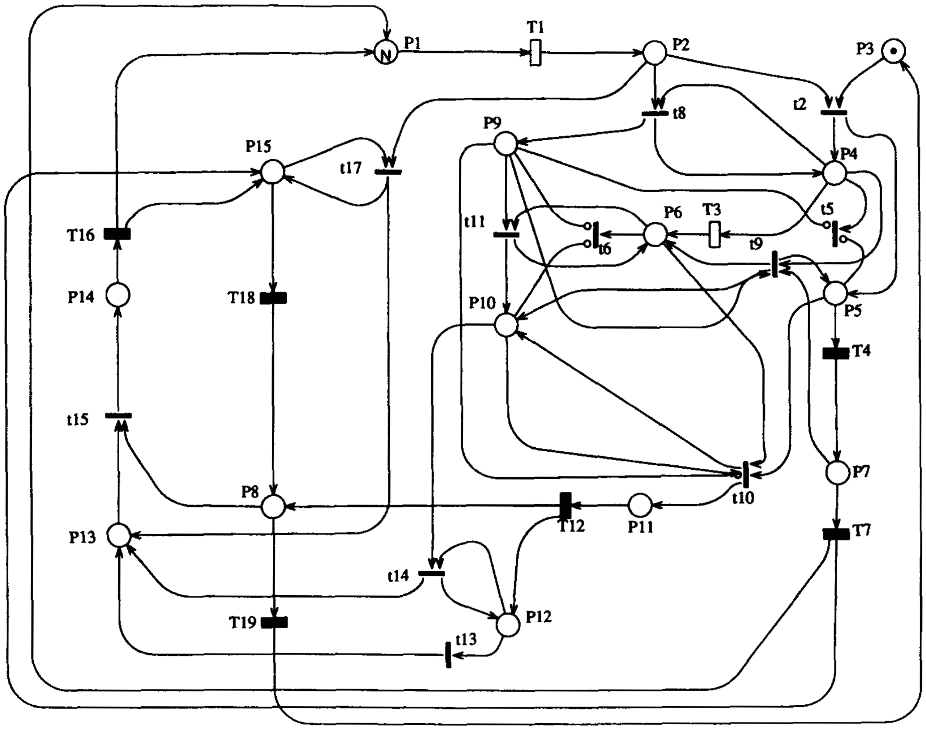


**Figure 3. DSPN model of CSMA/CD protocols with deterministic collision resolution**

mode as described below. In the marking which represents the vulnerable period of a transmission (places P4 and P5 contain a token) besides the competitively enabled timed transitions T3 and T4 the exponential transition T1 is concurrently enabled. Its firing in this marking represents a simultaneous transmission which causes a collision. This is represented by firing the immediate transition t8. Hence, the token in place P2 is moved to place P9 which disables the immediate transitions t5 and t6. After the firing of the timed transitions T3 or T4 the immediate transition t9 or t10 is enabled, respectively. The firing of either transition represents the end of the vulnerable period of a transmission in which a collision has occurred. The first token in place P9 is moved to place P11 representing the jamming delay after the detection of a collision. Possibly additional tokens in place P9 (further conflicting packets) are moved to place P10 by the immediate transitions t11. The deterministic transition T12 represents the jamming of stations after a collision. Its firing moves one token to place P8 and P12, respectively. This enables the immediate transition t14 which moves all tokens from place P10 (further stations involved in the collision) to P13. After the place P10 contains no more tokens, the immediate transition t13 fires and moves the token from place P12 (first conflicting station) to place P13. Then, the immediate transition t15 is enabled which models the start of the scheduling phase. To ensure the firing sequence of the immediate transitions t13, t14, and t15 the transitions t13 and t14 have associated the firing priority 2 and 3, respectively.

The following describes the DSPN subnet which represents a transmission in the reservation mode. Tokens in place P13 represent waiting stations with transmission requests. A token in place P14 models an ongoing transmission in the reservation mode. The overall transmission delay in the reservation mode is represented by means of the exponential transition T16. Since the scheduling delay depends on the number of ready stations, this transition is associated with a marking-dependent firing rate which corresponds to the reciprocal of the sum of the packet transfer time and the average duration of the scheduling phase in case of $i$ ready stations. After a transmission in the reservation mode a token is put back to place P1 (thinking stations) and a token is put into place P15. In case the place P2 contains one or more tokens (stations which have generated a packet during the last transmission phase) firing the immediate transition t17 moves all these tokens to place P13. Then, the deterministic transition T19 is enabled which models the delay for an interframe gap after a successful transmission. Its firing moves the token from place P15 to place P8 and another scheduling cycle begins. If the place P13 contains no token (no station has currently a ready packet) the deterministic transition T19 fires after the maximum scheduling delay. Its firing puts a token to place P3 which represents that the system has switched back to the contention mode. The marking-dependent firing delay associated with the deterministic transition T16 is given by.

$$\Psi(i) = 2\tau \cdot T(i_s) + T \qquad 1 \leq i \leq N \qquad (7)$$

In formula (7) the term $T(i_S)$ denotes the average number of slots required by the scheduling phase in case of *i* ready stations. The following shows how to derive the average sojourn times $T(i_S)$ for the CSMA/CD-BS and the CSMA/CD-LS protocol. The events [*i* stations are ready] and [*j* scheduling slots are required, i.e. the successful transmission starts in slot *(j+1)*] are denoted by [X = i] and [Y = j], respectively. The conditional probability for the event [Y = j assuming X = i] is specified as $P\{Y = j \mid X = i\}$.
We are assuming among all stations in the thinking state it is equally likely that a packet is generated. To simplify the analysis it is further assumed that in each selection phase the station numbers of ready stations are uniformly distributed among the set of feasible station numbers. The latter assumption clearly builds an approximation because ready stations closer in number to the last transmitting station have a higher probability to be selected in the scheduling phase. However, simulation experiments indicate that for the considered channel loads this approximation does not have a significant impact on the quantitative results. With these assumptions the mean value of a discrete cumulative distribution function which defines the $P\{Y = j \mid X = i\}$ for the CSMA/CD-BS protocol can be determined by the fraction of binomial coefficients.

To determine the denominator of this formula is straight-forward by employing a fundamental law from combinatorics which immediately yield the appropriate binomial coefficient [12]. The derivation of a closed-form expression for the numerator is a more complex task, though, already the special case of $N = 2^n - 1$ requires some analysis. In the following analysis we introduce $n = \lfloor \log_2 N \rfloor + 1$ for notational convenience. In a first step the case that there exists exactly one station with "0" from the highest to the (j-1)-th bit and with a "1" in the j-th bit. From combinatorics it is known that for $0 \leq j \leq n-1$ there exist $2^{n-j-1}$ such possibilities. Subsequently, the (i-1) other ready stations have a "0" from the highest to the j-th bit of the binary representation of its station number. The binomial coefficient corresponding to the number of such possibilities leads to the first expression in formula (8). As usual, binomial coefficients in which the upper term is smaller than the lower one or the lower term is negative are defined as zero. In the second expression all possibilities that two stations have the same bit pattern on the first (j-1) bits and differ only in the j-th bit are summed up. The inner sum represents the feasible permutations of the bit map of the binary representation of the station number of additional ready stations. Note, that for j=0 the sum indexed by ν is defined as zero.

$$\frac{\text{number of permutations of the bit map for which the binary maximum search requires } j \text{ steps}}{\text{number of ways to distribute } i \text{ entries among N slots}}$$

$$T(i_s) = \sum_{j=0}^{n-1} j \cdot \frac{2^{n-j-1} \cdot \binom{2^{n-j-1}-1}{i-1} + \sum_{\nu=1}^{2^{n-j-1}} \left\{ 2^{n-j-1} \cdot \binom{2^{n-j-1}}{\nu} \cdot \sum_{\mu=1}^{2^{j-1}} \binom{\mu 2^{n-j}-1}{i-\nu-1} \right\}}{\binom{N}{i}} \qquad (8)$$

The CSMA/CD-LS protocol [5] performs in the selection phase a linear minimum search among the station numbers corresponding to ready stations. With the assumptions mentioned above the mean value of the conditional discrete cumulative distribution function which underlies $P\{Y = j \mid X = i\}$ for the CSMA/CD-LS protocol is given by equation (9)

Again, the denominator of this formula represents the number of all feasible permutations. Thus, it is the same as in equation (8) for the case of the CSMA/CD-BS protocol. The closed-form expression for the numerator of equation (9) is also easily derived by fundamental combinatorics as the number of ways to distribute *(i-1)* entries among *(N-j-1)* slots.

$$T(i_s) = \sum_{j=0}^{N-i} j \cdot \frac{\binom{N-j-1}{i-1}}{\binom{N}{i}} \qquad (9)$$

## 4 Validation of the Modeling Approach

For a quantitative performance evaluation of the CSMA/CD protocols with limited contention we consider a local area network with a transfer rate of 10 Mbits/sec and a bus length of 1 km. A packet size of 1000 Bit is considered. Thus, the normalized propagation delay is given by $a=0.05$, respectively. Table 1 states the considered parameter setting of the DSPN model in case of the CSMA/CD-BS protocol. The basic time unit is chosen as 1 µsec. In case of the CSMA/CD-LS protocol everything stays the same except that the transition T19 has associated a delay corresponding to $2\tau N$.

| Trans. | Parameter | Value |
|---|---|---|
| T3 | propagation delay between pairs of stations | 3.546 |
| T4 | twice of the end-to-end propagation delay, $2\tau$ | 10 |
| T7 | remaining transfer time, $T - 2\tau$ | 90 |
| T12 | jamming delay, J | 15 |
| T16 | mean scheduling delay and transfer time | $\beta(i)^{-1} = 2\tau \cdot T(i_s) + T$ |
| T18 | interframe gap, f | 9.6 |
| T19 | maximum scheduling delay | $2\tau \cdot (\lfloor \log_2 N \rfloor + 1)$ |

**Table 1. Parameters of the DSPN model of the CSMA/CD-BS protocol**

As performance indices the average channel throughput and the average packet delay normalized to the packet transfer time, D, are considered. As in case of the compact DSPN model for the Ethernet [2] the channel throughput S is given by the forced-flow law. The average packet delay D can be determined by employing Little's law.

$$S = \lambda \cdot E\{\#P1\}$$

$$D = \frac{N - E\{\#P1\}}{\lambda \cdot E\{\#P1\}}$$

To validate the numerical results obtained by the DSPN model a simulation program for the CSMA/CD protocols with limited contention has been developed. For this task the modeling environment LANSF [8] has been employed which is particularly well suited for modeling medium access control protocols. The software package LANSF has been extended to generate confidence intervals for simulation output analysis. Programming components for discarding the initial transient by Schruben's test and for generating confidence intervals by non-overlapping batch means have been included in the software package LANSF according to their description in [11]. In Table 2 the throughput results of the DSPN model for the CSMA/CD-BS protocol are compared with the corresponding throughput values obtained by the LANSF simulation program for a LAN consisting of 60 stations. A confidence level of 99% and normalized propagation delays of a = 0.05 is considered. The validation of the throughput results of the DSPN with deterministic transition T16 yield excellent agreement with corresponding simulation results. This holds for the DSPN with exponential transition T16 for low and middle loads, too. For heavy loaded systems the approximate representation of the reservation phase by an exponentially distributed delay introduces some error to the throughput. Similar results have been obtained for the CSMA/CD protocol with linear search.

| G | Confidence intervals for throughput of LANSF simulation | DSPN with marking-dependent exponential transition T16 | DSPN with marking-dependent deterministic transition T16 |
|---|---|---|---|
| 0.125 | [0.123, 0.130] | 0.125 | 0.126 |
| 0.250 | [0.242, 0.256] | 0.249 | 0.251 |
| 0.375 | [0.365, 0.382] | 0.371 | 0.375 |
| 0.500 | [0.477, 0.503] | 0.490 | 0.493 |
| 0.625 | [0.596, 0.624] | 0.602 | 0.612 |
| 0.750 | [0.703, 0.729] | 0.690 | 0.718 |
| 0.875 | [0.727, 0.743] | 0.723 | 0.737 |
| 1.000 | [0.696, 0.702] | 0.687 | 0.700 |
| 1.250 | [0.675, 0.677] | 0.658 | 0.677 |
| 1.500 | [0.666, 0.667] | 0.645 | 0.667 |
| 2.000 | [0.659, 0.660] | 0.641 | 0.660 |

**Table 2. Comparison of throughput results for a = 0.05**

## Conclusions

In this paper the association of a marking-dependent firing delay to a deterministic transition of a DSPN has been proposed. This extension of the original DSPN modeling definition [2] allows a compact representation of deterministic medium access schemes. In general, additional semantics have to be considered to properly define the association of marking-dependent firing delays to deterministic transitions [1]. Thus, we have assumed that the firing delay of a deterministic transition is specified by the marking in which it has become enabled. Formulas for calculating a row of the state transition matrix P and the conversion matrix C of the embedded Markov chain of a DSPN have been presented for this case.

A DSPN model for CSMA/CD protocols with deterministic collision resolution has been introduced. The novel feature of the DSPN model lies in the aggregated representation of the reservation mode by means of one deterministic transition with a marking-dependent firing delay. Since the marking condition remains constant during enabling interval of the deterministic transition, no additional semantics have to be considered for specifying the marking-dependent firing delay. The cardinality of the state space of this DSPN grows only quadratically with increasing number of stations. Thus, the DSPN model can effectively employed for evaluating LANs with a large number of stations (e.g. a LAN with 60 stations). Moreover, the association of a marking-dependent firing delay to a single timed transition has the advantage that the graphical representation of the DSPN is independent from the number of stations of the LAN.

Further research is concentrating on employing the presented approach for evaluating the performance of medium access control protocols for high-speed LAN's.

## Acknowledgments

The author is grateful to Adam Wolisz for various helpful discussions about CSMA/CD protocols with deterministic collision resolution.

## References

[1] M. Ajmone Marsan, G. Balbo, A. Bobbio, G. Chiola, G. Conte, A. Cumani, "The Effect of Execution Policies on the Semantics of Stochastic Petri Nets", *IEEE Trans. Softw. Engin., 15*, pp. 832-845, 1989.

[2] M. Ajmone Marsan, G. Chiola, and A. Fumagalli, "An Accurate Performance Model of CSMA/CD Bus LAN", in: *G. Rozenberg (Ed.) Advances in Petri Nets 1986, Lecture Notes in Computer Science 266*, pp. 146-161, Springer 1987.

[3] M. Ajmone Marsan and G. Chiola, "On Petri Nets with Deterministic and Exponentially Distributed Firing Times", in: *G. Rozenberg (Ed.) Advances in Petri Nets 1986, Lecture Notes in Computer Science 266*, pp. 132-145, Springer 1987.

[4] J.I. Capetanakis, "Generalized TDMA: The Multi-Accessing Tree Protocol", *IEEE Trans. on Comm.*, 27, pp. 1476-1484, 1979.

[5] J.S.J. Chen and V.O.K. Li, "Reservation CSMA/CD: A Multiple Access Protocol for LAN's", *IEEE Journal on Select. Areas in Comm., 7*, pp. 202-210, 1989.

[6] G. Chiola, "A Graphical Petri Net Tool for Performance Analysis", *Proc. 3rd Int. Conf. on Modeling Techniques and Tools for Performance Analysis, Paris France*, pp. 323-333, 1987.

[7] I. Chlamtac, W.R. Franta, and K.D. Levin, "BRAM: The Broadcast Recognition Access Method", *IEEE Trans. on Comm.*, 27, pp. 1183-1189, 1979.

[8] P. Gburzynski, P. Rudnicki, "LANSF: A Protocol Modelling Environment and its Implementation", *Software - Practise and Experience, 21*, pp. 51-76, 1991.

[9] C. Lindemann, "An Improved Numerical Algorithm for Calculating Steady-state Solutions of Deterministic and Stochastic Petri Net Models", *Proc. 4th Int. Workshop on Petri Nets and Performance Models, Melbourne Australia*, pp. 176-185, 1991.

[10] J. Mierzwa and A. Wolisz, "A Multiple Random Access Protocol for Bus Structured Local Area Networks Operating in Real-Time Environments", *Proc. 9th Triennial World Congress of IFAC, Budapest Hungary*, pp. 2713-2717, 1984.

[11] K. Pawlikowsky, "Steady-State Simulation of Queueing Processes: A Survey of Problems and Solutions", *ACM Computing Surveys, 22*, pp. 124-169, 1990.

[12] K.S. Trivedi, *"Probability and Statistics with Reliability, Queuing, and Computer Science Applications"*, Prentice Hall 1982.